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A Study on Performance Characteristics of Granular-Media Trickling Filters

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ABSTRACT

The performance characteristics of the unsaturated granular media were described using Monod type biological kinetics and mass transfer concepts within microbial films. For this purpose computer techniques were first developed for the numerical evaluation of the normalized biofilms mathematical model. The effects of wetted surface area and the other parameters on substrate removal were then numerically evaluated. The theory enables one to transfer the experimental findings obtained from a pilot plant to another filter for design purposes. Practical application of the theory to experimental results was also demonstrated.

Index Entries: Biological surveys; concentrations; design practice; diffusion; environmental engineering; numerical methods; mass transfer; mathematical models; parameters; slime; substrates; temperature effects; trickling filters.

NOMENCLATURE

A = cross-sectional area, L^2
B_i = dimensionless substrate concentration
B_{si} = dimensionless substrate concentration at the liquid-biofilm interface

$\overline{B}_{so}$ = lower limit in the definite integral $\overline{\overline{Z}}$ calculated using Eq. 27
$\overline{B}_o$ = dimensionless effluent substrate concentration corresponding to $\overline{B}_{so}$
b = total width of parallel planes in conceptual model, L
c = bulk liquid phase substrate concentration, ML^{-3}
c_o = bulk liquid phase effluent substrate concentration, ML^{-3}
c_s = substrate concentration at the liquid-biofilm interface, ML^{-3}
c_x = substrate concentration at any location x, ML^{-3}
c_i = bulk liquid phase inlet substrate concentration, ML^{-3}
C = dimensionless substrate concentration defined as c/c_i
C_x = dimensionless substrate concentration defined as c_x/c_i
D_c = diffusivity of substrate in biofilm, $L^2 T^{-1}$
D_w = molecular diffusivity of substrate in the liquid, $L^2 T^{-1}$
E = mass transfer coefficient defined as D_w/e, LT^{-1}
e = depth of stagnant liquid layer adjacent to biofilm, L
F = mathematical expression to calculate the definite integral $\overline{\overline{Z}}$
H = depth of filter, L
h = liquid film thickness, L
K = dimensionless ratio of mass transfer rate to kinetic rate
K_s = Monod-half velocity coefficient, ML^{-3}
k = maximum utilization rate of rate limiting substrate, T^{-1}
k_1 = biological rate (equation) coefficient T^{-1}
k_2 = biological rate (equation) coefficient L^{-1}
k_3 = biological rate (equation) coefficient $M^{-1} L^3$
L = wet microbial film thickness, L
ℓ = dimensional filter length, L
M = dimensionless biofilm thickness
N = flux of substrate, $ML^{-2} T^{-1}$
Q_A = hydraulic loading rate, $L^3 L^{-2} T^{-1}$
q = rate of flow per unit width, $L^3 T^{-1} L^{-1}$
Q = volumetric rate of flow, $L^3 T^{-1}$
S = specific surface area, $L^2 L^{-3}$
w_{av} = average velocity of liquid in z direction, LT^{-1}
Z = dimensionless distance measured in flow direction from the origin
$\overline{\overline{Z}}$ = value of the definite integral related to the filter length
z = axial distance measured in flow direction from the origin, L
α = dimensionless filter depth
X = dimensionless distance measured normal to flow direction
x = dimensional distance measured normal to flow direction, L
X_c = microbial density within biofilm, ML^{-3}
γ = specific gravity, $ML^{-2} T^{-2}$
μ = dynamic viscosity, $ML^{-1} T^{-1}$
λ = effectiveness coefficient
ρ = mass density of liquid, ML^{-3}
η = biological removal ratio (biological efficiency)
η_D = $k_s h / D_w$ in which k_s is a proportionality constant in LT^{-1}

INTRODUCTION

An understanding of the factors affecting the rates of biofilm reactions is essential in designing waste water treatment processes. Attached growth biological treatment systems have been widely used in the application of waste water technology (*1,2*).

In a previous paper (3), a hydraulically-controlled uniform biofilm was taken into consideration. The microbial film is covered with a liquid film, of which wetted surface depends upon the flow rate and the media characteristics. A conceptual model consisting of parallel planes covered with a biofilm was used to study the substrate utilization.

A mass balance equation was written using Monod kinetics at any point within the biofilm. The most convenient form of solution to this equation was obtained when the mathematical problem was expressed in dimensionless terms. The following normalizing parameters were used (*4–6*).

$$f = c_f / cs \; ; k_l = kX_c / K_s \; ; M = [kX_c / D_cK_s]^{1/2} \; L = k_2L \; ; X = x / L \qquad (1)$$

$$k_2 = [kX_c / D_cK_s]^{1/2} \; ; B = c_f / K_s = k_3c_f \; ; k_3 = 1 / K_s \qquad (2)$$

in which c_f = substrate concentration within the biofilm, ML^{-3}; c_s = substrate concentration at liquid biofilm interface, ML^{-3}; $f = c_f/c_s$ = dimensionless substrate concentration inside the biofilm; X = dimensionless distance in x-direction; k = maximum utilization rate of the rate limiting substrate, T^{-1}; k_1 = a biological rate equation coefficient, T^{-1}; K_s = Monod-half velocity coefficient, ML^{-3}; L = planar bacterial film thickness, L; B = a dimensionless substrate concentration inside the liquid film; X_c = bacterial density within biofilm, ML^{-3}; D_c = the diffusivity of the substrate in the biofilm, L^2T^{-1}; M = a dimensionless microbial film thickness; k_2 = a coefficient related to a solid phase diffusional limitation, L^{-1}; k_3 = a biological rate equation coefficient, $M^{-1}L^3$ (*see* Fig. 1).

For a given dimensionless filter depth α, the dimensionless substrate concentration is reduced from an initial value of B_{si} (or B_i) to an effluent substrate concentration of B_{so} (or B_o). The corresponding substrate removal efficiency $\eta = 1 - (B_o/B_i)$ is a function of inlet substrate concentration B_i (or c_i). The values of η were obtained from a computer program using the following input data:

$$K = Ek_2 / k_1 \; ; \alpha = k_1L / k_2L \, \ell / q \qquad (3)$$

$$A_o = k_2 / k_1 q \; ; A_1 = 10^{-6}k_3 \qquad (4)$$

$$E = D_w / e \qquad (5)$$

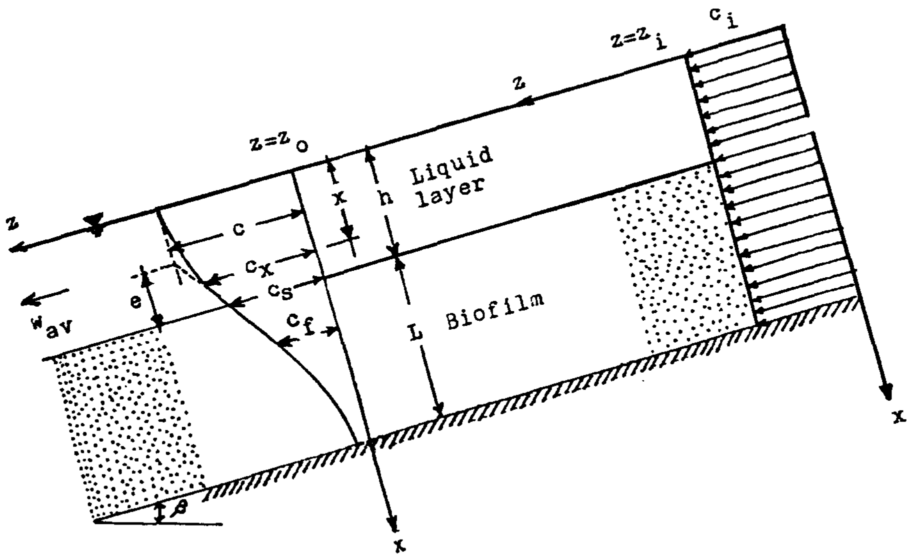

Fig. 1. Schematic of substrate distribution in inclined plane model of biofilm.

in which E = mass transfer coefficient, LT^{-1}; e = depth of stagnant liquid layer adjacent to biofilm, L; ℓ = filter length, L; q = rate of flow per unit width, $L^3T^{-1}L^{-1}$ and D_w = molecular diffusivity of substrate in the liquid film, L^2T^{-1}. K is a dimensionless ratio of mass transfer rate to kinetic rate. A_o, A_1, α, and M are computer input data as defined above.

Differential equation resulting from a mass balance inside the liquid layer was solved for the special case of low concentration asymptote. In this case the ratio of substrate uptake N becomes (*6–9*):

$$N = k_s c_s \tag{6}$$

in which

$$k_s = k_1 L \quad \text{for thin biofilms} \tag{7}$$

$$k_s = k_1 / k_2 \quad \text{for thick biofilms} \tag{8}$$

The following parameters were used to normalize the dispersion equation:

$$Z_D = (2/3)(D_w/h)(\ell/q) \tag{9}$$

$$\eta_D = k_s h / D_w \tag{10}$$

in which h = liquid film thickness, L. The value of e in Eq. 5 to calculate the mass transfer coefficient E were thus obtained. The resulting values of e/h were given in Table 1 as a function of η_D and Z_D (*3,8*).

The most satisfactory method of determining biological parameters k_1, k_2, and k_3 is to use the asymptotic expression together with the experimental data obtained from a flat plate biological film reactor because the hydraulic conditions are clear and the physical parameters can easily

Table 1
Ratios of e/h for Different Values
of η_D and Z_D to Calculate the Mass Transfer Coefficient of E

	η_D											
Z_D	0.05	0.10	0.20	0.50	0.75	1.00	2.00	4	6	8	10	100
0.01	0.250	0.251	0.261	0.265	0.267	0.262	0.268	0.276	0.281	0.285	2.88	0.311
0.02	0.308	0.309	0.320	0.313	0.315	0.323	0.331	0.341	0.347	0.352	0.355	0.370
0.04	0.359	0.372	0.377	0.383	0.387	0.389	0.399	0.411	0.418	0.423	0.426	0.442
0.06	0.403	0.408	0.414	0.418	0.426	0.427	0.438	0.450	0.457	0.461	0.464	0.480
0.08	0.424	0.431	0.436	0.444	0.448	0.451	0.462	0.474	0.480	0.484	0.487	0.499
0.10	0.438	0.446	0.451	0.458	0.462	0.466	0.477	0.488	0.494	0.497	0.500	0.510
0.12	0.449	0.456	0.461	0.469	0.473	0.476	0.486	0.497	0.502	0.505	0.507	0.517
0.14	0.449	0.463	0.468	0.475	0.479	0.482	0.492	0.502	0.507	0.510	0.512	0.520
0.16	0.449	0.467	0.472	0.479	0.483	0.487	0.496	0.505	0.510	0.512	0.514	0.522
0.18	0.449	0.470	0.475	0.482	0.486	0.489	0.498	0.507	0.511	0.514	0.516	0.523
0.20	0.449	0.472	0.477	0.484	0.487	0.491	0.500	0.508	0.512	0.515	0.516	0.523
0.26	0.449	0.473	0.480	0.486	0.490	0.493	0.501	0.509	0.513	0.516	0.517	0.523
0.30	0.449	0.476	0.480	0.487	0.490	0.494	0.502	0.510	0.514	0.516	0.517	0.523
0.40	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523
0.50	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523
1.00	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523
1.50	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523
2.00	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523
2.50	0.449	0.476	0.481	0.487	0.491	0.494	0.502	0.510	0.514	0.516	0.517	0.523

be defined. When c_i tends to zero, the general equation of substrate utilization reduces to (*7–9*).

$$\ln c_o / c_i \cong - [K \tanh M / (K + \tanh M)] \cdot \alpha \tag{11}$$

For thin biofilms ($M < 0.5$), when $c_i \rightarrow 0$, Eq. 11 becomes

$$\ln c_o / c_i \cong - [K / (K/M + 1)] \cdot \alpha \tag{12}$$

On the other hand, for all values of L, when c_i is sufficiently large, the general equation of substrate utilization reduces to:

$$\eta \cong 1 - (c_o / c_i) = (k_1 L / k_3)(\ell / q)(1 / c_i) = (I)(1 / c_i) \tag{13}$$

in which

$$I = (k_1 L / k_3)(\ell / q) \tag{14}$$

If the experimental data are accordingly replotted in terms of η vs ($1/c_i$), the result is a linear relationship at high concentrations with a slope of I.

STRUCTURE OF GRANULAR MEDIA

Due to the complex nature of granular-media, conceptual models are used to study the flow phenomena. Consider an element of filter bed

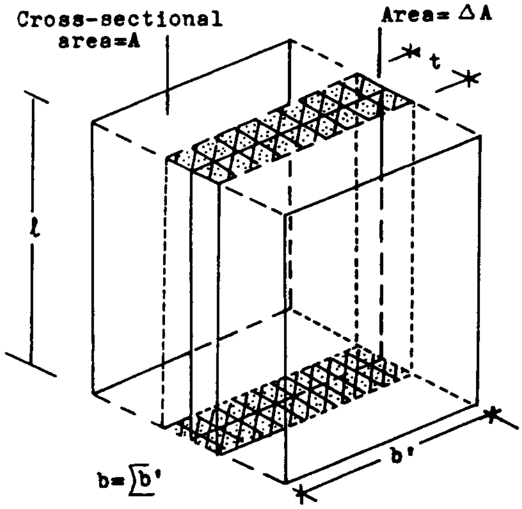

Fig. 2. Conceptual model to study biological filtration.

with a cross sectional area ($\triangle A$) and depth ℓ. The cross sectional area of the filter bed is denoted by A (Fig. 2). If the wetted specific surface area of the filter medium is S_w, then the wetted surface area of the filter bed is given by $S_w (A\,\ell)$ which should be equal to the wetted area in the conceptual model (*10,11*).

$$b\ell = S_w(A\,\ell) \quad (15)$$

in which b denotes the wetted perimeter (width of the section under consideration). Hence

$$b = S_w A \quad (16a)$$

If the cross sectional area ($\triangle A$) of the element of the filter bed includes only one vertical plate as shown in Fig. 2, Eq. 16a becomes

$$b' = S_w (\triangle A) = S_w b' t \quad (16b)$$

in which b′ is the width of each vertical plate, t is the distance between two parallel plate and $\triangle A = b't$. Eq. 16b leads to

$$S_w = 1 / t \quad \text{or} \quad t = 1 / S_w \quad (16c)$$

These equations convert the filter media into a series of parallel planes of which total width is equal to $b = \Sigma b'$. Therefore, the following equations can be written for flow rate per unit width q

$$q = Q / b = Q/(S_w A) = Q/A / S_w = Q_A/S_w \quad (17)$$

in which Q is the volumetric flow rate in L^3T^{-1} passing through a cross sectional area A of a filter bed and Q_A is the hydraulic loading rate in LT^{-1}.

The total surface area of grains per unit volume of packing is defined as the specific surface area S. This definition is for a dry filter, i.e., before the liquid application. If the wetted area is considered, this definition should be modified to give the wetted specific surface area S_w mentioned before. The wetted area would be expected to vary with the flow rate. Using a proportionality constant Ψ, the wetted specific surface area S_w can be defined based upon Ψ and S as follows

$$S_w = \Psi S \tag{18}$$

Substitution of Eq. 18 into Eq. 17 results in

$$q = Q_A / S_w = Q_A / \Psi S \tag{19}$$

Equation 19 can now be combined with the equations previously developed. For example, from Eqs. 3 and 19 it results

$$\alpha = (k_1L / k_2L)(\ell / q) = (k_1L / k_2L)(\ell \Psi S / Q_A) = (k_1L / k_2L)(\Psi S\ell / Q_A) \tag{20}$$

In a similar way Eq. 14 becomes

$$I = \eta c_i = c_i - c_o = (k_1L / k_3)(\ell / q) = (k_1L / k_3)(\ell \Psi S / Q_A) = (k_1L / k_3)(\Psi S\ell / Q_A) \tag{21}$$

Dimensionless filter depth α can now be obtained substituting $S\ell/Q_A$ from Eq. 21 into Eq. 20 as

$$\alpha = (k_1L / k_2L)(\Psi I k_3 / k_1L)(1 / \Psi) = I k_3 / k_2L \tag{22}$$

The value of Ψk_1L is an important characteristic of the wetted surface. Its value can now be derived from Eq. 21 as

$$\Psi k_1L = I k_3 Q_A / S\ell \tag{23}$$

VARIATION OF WETTED SURFACE AREA WITH FLOW RATE

Process characteristics that may be studied include the substrate concentration, flow rate, and temperature for a given reactor system. The effect of concentration has already been studied. The effect of temperature will be the subject of a separate paper. The effect of flow rate was investigated below.

The variation of the wetted surface area of grains with the rate of flow was first studied for this purpose. The conceptual model described converts the filter into a series of parallel planes. Using the parabolic velocity profile of the steady uniform laminar flow over the inclined plane, the average velocity is obtained from (*10–12*)

$$w_{av} = (\gamma \sin \beta / 3 \mu) h^2 \tag{24}$$

in which $\sin\beta$ = slope of inclined plane; μ = the dynamic viscosity, $ML^{-1}T^{-1}$, and γ = the specific gravity, $ML^{-2}T^{-2}$. The flow rate of per unit width is given by

$$q = (\gamma \sin\beta / 3\mu)\, h^3 = m\, h^3 \tag{25}$$

in which

$$m = \gamma \sin\beta / 3\mu$$

or

$$h = [3\mu / \gamma \sin\beta]^{1/3}\, q^{1/3} = n\,(Q_A / S_w)^{1/3} \tag{26}$$

in which

S_w = wetted specific surface = wetted area per unit volume of packing, L^{-1}
Q_A = hydraulic loading rate, LT^{-1} and
n = $[3\mu / \gamma \sin\beta]^{1/3}$

Now express the wetted specific surface area S_w using proportionality constant Ψ. Substituting Eq. 19 into Eq. 25 leads to

$$q = Q_A / S_w = Q_A / \Psi S = mh^3 \tag{27}$$

or

$$Q_A / S = \Psi\, mh^3 \tag{28}$$

In the following discussion Q_A/S will be denoted with Q_p, namely as

$$Q_p = Q_A / S \tag{29}$$

The values of m and S in Eq. 28 are constant in a given filter bed. Therefore, if h and Ψ are known for a hydraulic loading rate $(Q_A)_1$, the corresponding parameters for a hydraulic loading rate $(Q_A)_2$ can then be obtained from

$$(Q_p)_1 / (Q_p)_2 = [(\Psi)_1 / (\Psi)_2]\, [(h)_1^3 / (h)_2^3] \tag{30}$$

in which the subscripts written outside the brackets indicate the corresponding flow rates. In order to obtain a general relationship for the variation of the wetted surface area with flow rate, the following assumption was made and experimentally confirmed in this study:

$$(\Psi)_1 / (\Psi)_2 = (h)_1 / (h)_2 \tag{31}$$

Substitution of Eq. 31 into Eq. 30 results in

$$(Q_p)_1 / (Q_p)_2 = (\Psi)_1^4 / (\Psi)_2^4 \tag{32}$$

or

$$(h)_1 / (h)_2 = (\Psi)_1 / (\Psi)_2 = [(Q_p)_1 / (Q_p)_2]^{1/4} \tag{33}$$

Average velocity w_{av} is obtained from the unit width discharge as

$$w_{av} = q / [(1)(h)] = Q_A / [(S_w)(h)] = Q_A / (\Psi Sh) = Q_p / (\Psi h) \tag{34}$$

Velocity ratio then becomes

$$(w_{av})_1 / (w_{av})_2 = [(Q_P)_1 / (Q_P)_2] [(\Psi)_2 / (\Psi)_1] [(h)_2 / (h)_1] = [(Q_P)_1 / (Q_P)_2]^{1/2} \quad (35)$$

THE EFFECT OF FLOW RATE ON SUBSTRATE REMOVAL EFFICIENCY

All Parameters Except the Hydraulic Loading Rate Are Constant

Now calculate the relation between the biological efficiency η and the inlet substrate concentration c_i for a flow rate $(Q_P)_2$, when this relationship is known for a flow rate $(Q_P)_1$, the other variables such as ℓ and S including liquid temperature being same for these flow rates. The parameters $(Z_D)_1$ and $(\eta_D)_1$ are also known for the rate of flow $(Q_P)_1$ because $(Z_D)_1$ is calculated using Eq. 9 and $(\eta_D)_1$ is obtained from

$$(\eta_D)_1 = [k_1L] [(h)_1 / (D_w)_1] \quad \text{for thin biofilms} \quad (36)$$

$$(\eta_D)_1 = [k_1L / k_2L] [(h / D_w)_1] \quad \text{for thick biofilms} \quad (37)$$

In order to find the $\eta \sim c_i$ relationship for the flow rate $(Q_P)_2$, the values of $(Z_D)_2$ and $(\eta_D)_2$ should first be determined. These parameters being known, the ratio of e/h is then obtained from Table 1 (3). Thus the value of K and the other data necessary for the computer program are calculated using Eqs. 3–5 as explained below.

In order to determine the parameters $(Z_D)_2$ and $(\eta_D)_2$, Eqs. 36 or 37 is once more written for the rate of flow $(Q_P)_2$. The biological parameters k_1L and k_2L as well as D_w do not change with rates of flow. Therefore, when divided sides by sides, they cancel each other and the substitution of Eq. 33 into the resulting equation leads to

$$(\eta_D)_2 = (\eta_D)_1 [(Q_P)_2 / (Q_P)_1]^{1/4} \quad (38)$$

Using Eqs. 25 and 33 one obtains from Eq. 9

$$(Z_D)_2 / (Z_D)_1 = [(h)_1 / (h)_2] \cdot [(q)_1 / (q)_2] = [(h)_1 / (h)_2]^4 \quad (39)$$

In a similar way the following ratio can be written

$$(D_w/h)_2 / (D_w/h)_1 = (h)_1 / (h)_2 = [(Q_P)_1 / (Q_P)_2]^{1/4} \quad (40)$$

All Parameters Except the Liquid Temperature Are Changing

The similar relationships can be derived when the specific surface area S and the filter depth ℓ are not same for the given hydraulic loading rates $(Q_A)_1$ and $(Q_A)_2$. The results of the two different filters can thus be compared for the same substrate. In this case Eqs. 38 and 40 remain unchanged. The other equations, however, should be modified as follows, e.g., Eq. 39 becomes

$$(Z_D)_2 / (Z_D)_1 = [(h)_1 / (h)_2] [(q)_1 / (q)_2] [(\ell)_2 / (\ell)_1] \quad (41)$$

in which $(\ell)_2$ is the depth of the filter treating the same substrate with a hydraulic loading rate of $(Q_A)_2$. The filter depth was denoted with $(\ell)_1$ for the hydraulic loading rate $(Q_A)_1$. Substitution of Eq. 25 into Eq. 41 leads to

$$(Z_D)_2 / (Z_D)_1 = [(h)_1 / (h)_2]^4 [(\ell)_2 / (\ell)_1] \quad (42)$$

Introducing Eq. 33 into Eq. 42 results in

$$(Z_D)_2 = (Z_D)_1 [(Q_P)_1 / (Q_P)_2] [(\ell)_2 / (\ell)_1] \quad (43)$$

COMPARISON OF THE PERFORMANCES OF DIFFERENT FILTERS FOR THE SAME SUBSTRATE

The factors that control the conversion efficiency of a filter can be divided into two main groups, i.e., biological and physical (Atkinson and Abdel Rahman Ali, 1976) (*13,14*). The former is covered by the kinetic coefficients of the microbe substrate system (k_1, k_2, and k_3). The kinetic co-coefficients are same for all filters treating the same substrate. The physiical factors comprise the size of the bed, specific surface area of the packing, and the liquid phase mass transfer characteristics. If the effect of these parameters on substrate removal is known, the experimental results obtained from a pilot plant treating this substrate can be transferred to an actual filter to predict the substrate removal. For this purpose, equations derived in the preceding sections can be used. In order to demonstrate the practical application of the theory, the data published by Atkinson and Williams (1971) (*5*) and later by Atkinson and Abdel Rahman Ali (1976) (*15*) were evaluated. In these experiments a packed bed of wooden spheres with a diameter of 2 in (=5.08 cm) was used to develop a thick microbial film. In the experiments of Atkinson and Williams (1971) (*5*), the filter bed consisted of three pieces and its total depth was 2 ft (=60.96 cm). The microbial film thickness was controlled by hydraulic washing. The specific surface area and the porosity of filter were reported as $S=0.712$ cm^{-1} and $\epsilon=0.406$, respectively. The liquid was applied to the filter by means of a hydraulic circuit in which feed solution was automatically prepared in a large storage tank by mixing a concentrated glucose solution with mains water. Feed solution was enriched with nitrogen and phosphorous. Inlet and outlet glucose concentrations and the liquid flow rate were measured under the condition of constant temperature (=20°C). Filter efficiencies η for hydraulic loading rates of

$Q_A = 0.0148$; 0.027; 0.054; and 0.081 $cm^3cm^{-2}s^{-1}$ were calculated from these measured values (*16*). They were plotted against the inlet substrate concentrations c_i (*16*). The input data for the computer program were found based upon the asymptotic conditions for each hydraulic loading rate. The value of $k_3 = (1.706)(10^5)\ cm^3g^{-1}$ obtained by Atkinson and Daoud (1970) (*4*) from thin biofilm experiments for the same glucose feed solution was used in this investigation. Using these input data and the computer program developed, substrate removal efficiencies η were calculated and plotted against c_i (*16*). Good agreement was obtained between the measured and predicted values. The computer input data obtained for 20°C were recalculated for 25°C. These calculations and the evaluation of the experimental results to determine the biological system parameters will be discussed elsewhere. Therefore only the results are given in this paper. These values are used in the following discussion as a basis to study the effect of the physical parameters on removal efficiencies. The resulting values for a hydraulic loading rate of $Q_A = 0.0148\ cm^3cm^{-2}s^{-1}$ are given below as an example:

$$\Psi k_1 L = 0.0044873\ cm^2s^{-1}cm^{-3} \tag{44a}$$

$$M = k_2L = 8.4909\ ;\ k_3 = (1.706)(10^5)\ cm^3g^{-1} \tag{44b}$$

$$\alpha = 1.5499\ ;\ K = 6.40877 \tag{44d}$$

$$A_1 = 0.1706\ cm^3g^{-1}\ ;\ D_w / h = 0.0016359 / \Psi \tag{44d}$$

$$E = D_w / e = 0.003387 / \Psi\ ;\ \eta_o = 0.32306 \tag{44e}$$

$$Z_D = 3.19846\ ;\ e / h = 0.483 \tag{44f}$$

The graphs of η vs c_i (or ηc_i vs c_i) can be replotted for any length, specific surface area and flow rate. This procedure is explained below based upon the input data given above T=25°C and $Q_A = 0.0148\ cm^3cm^{-2}s^{-1}$ as an example.

EXPERIMENTAL CONFIRMATION

The validity of the theoretical model was demonstrated by experiments using the same synthetic feed solution as Atkinson and Williams (1971) (*5*). Basically, the apparatus consisted of a packed bed of plastic spheres of 36.8 mm diameter, upon which a microbial layer was developed in response to a continuous liquid feed containing glucose and other nutrients. The microbial film thickness was controlled by hydraulic washing. The packed bed was contained in a cylinder of 35.4 cm diameter. The depth of the filter was L=165 cm, its porosity was $\epsilon = 0.408$ and the specific surface of the medium was $S = 0.966\ cm^2cm^{-3}$. The feed solution was prepared in a large storage tank by mixing a concentrated glucose solution and

Table 2
Filter Performance Under Various Experimental Conditions[a]

Run	Rate flow Q cm^3s^{-1}	Measured COD Influent c_i mg dm^{-3}	Effluent c mg dm^{-3}	Measured COD removal efficiency $\eta = 1 - c_o/c_i$	Flow rate per unit width $Q_P = Q_A / S$ $cm^3cm^{-2}s^{-1}$	Hydraulic loading rate Q_A $cm^3cm^{-2}s^{-1}$
1	0.962	624.0	108.5	0.826	0.00101	0.000968
2	1.840	949.5	208.0	0.781	0.00194	0.001874
3	4.260	813.9	271.3	0.666	0.00449	0.004337
4	4.950	840.9	307.5	0.634	0.00521	0.005033
5	5.710	569.7	244.2	0.571	0.00601	0.005806
6	9.900	748.8	403.2	0.462	0.01040	0.010050
7	10.200	601.6	345.9	0.425	0.01070	0.010340
8	14.280	597.6	374.4	0.373	0.01500	0.014470
9	25.640	504.0	345.6	0.314	0.02700	0.026080
10	62.500	563.8	450.8	0.200	0.06580	0.063560
11	74.070	546.0	452.4	0.171	0.07800	0.075350
12	82.640	592.8	478.8	0.190	0.08700	0.084000

[a]Depth of filter $\ell = 165$ cm; diameter of filter, D = 35.4 cm; liquid temperaure = 25°C; specific surface, S = 0.966 cm^2cm^{-3}; porosity, $\epsilon = 0.408$.

nutrients with tap water. Stock glucose-nutrient feed solution was prepared once a week and refrigerated during the experimental runs. The feed solution was pumped to a constant head tank. From there, it was metered by Rotameter, passed to a distributor and applied to the filter. Once the operating conditions had been adjusted to the desired values, the appropriate data were collected. These data consisted of inlet and outlet COD concentrations, the liquid flow rates and the temperature of the feed solution (Table 2). The performance of the filter was investigated at flow rates of 0.962–82.64 cm^3s^{-1}. The efficiency of the filter was measured in terms of the chemical oxygen demand (COD) of the solution. COD concentration was varied over the range of 504–840.9 mg dm^{-3}.

In the data presented by Atkinson and Williams (1971) (*5*), the results were expressed in terms of glucose concentrations, whereas in this experiment η values were measured as COD concentrations. In order to compare the results of both filters, the COD concentrations should be converted to glucose concentrations. For this purpose, theoretical oxgyen demand (ThOD) for glucose was used

$$C_6H_{12}O_6 + 6\,O_2 \rightarrow 6\,CO_2 + 6\,H_2O \tag{45}$$

$$\begin{aligned} \text{ThOD} &= 6 \text{ moles of } O_2/\text{mole of glucose} \\ &= 192 \text{ g}O_2/180 \text{ g glucose} \cong 1.07 \text{ g } O_2/\text{g glucose} \\ &= 1.07 \text{ mg } O_2/\text{mg dm}^{-3} \text{ glucose} \end{aligned}$$

If a COD/ThOD ratio of (1.0/1.07) is assumed, COD concentration becomes equal to glucose concentration in mg dm^{-3}.

Effluent concentrations for this filter can be predicted using the parameters obtained before from the filter of Atkinson and Williams (1971) (3) for $(Q_A)_1 = 0.0148\ cm^3cm^{-2}s^{-1}$ (*see* Eq. 44). For this purpose, the input data for this filter should first be calculated and the results obtained from the computer program should be compared with the measured values written in Table 2. This comparison will be made for the hydraulic loading rate $(Q_A)_2 = 0.005033\ cm^3cm^{-2}s^{-1}$ (run No. 4 in Table 2) for purpose of illustration. The input data for the computer program for this hydraulic loading rate can be obtained multiplying the corresponding values of Atkinson and Williams (1971) (5) by the ratios calculated before. In this case $(Q_A)_2 = 0.005033\ cm^3cm^{-2}s^{-1}$; $(Q_P)_2 = (Q_A/S)_2 = 0.005033/0.966 = 0.005210\ cm^3cm^{-1}s^{-1}$ and $(\ell)_2 = 165$ cm, whereas the corresponding values in the Atkinson and Williams (1971) (5) are $(Q_A)_1 = 0.0148\ cm^3cm^{-2}s^{-1}$; $(Q_P)_1 = 0.0148/0.712 = 0.020787\ cm^3cm^{-1}s^{-1}$ and $(\ell)_1 = 60.96$ cm. From Eq. 43 and Eq. 44f:

$$(Z_D)_2 = (3.19846)\,(0.020787 / 0.00521)\,(165.00 / 60.96) = 34.541$$

From Eq. 38 and Eq. 44e

$$(\eta_D)_2 = 0.32306\,[0.00521 / 0.020787]^{1/4} = 0.2286$$

Table 1 gives a value of $(e/h)_2 = 0.481$ for the values of $(\eta_D)_2 = 0.2286$ and $(Z_D)_2 = 34.541$ calculated above. In a similar way, the values of D_w/h and E can be obtained from Eqs.40 and 44d:

$$(D_w / h)_2 = [0.0016359 / (\Psi)_{14.8}]\,[0.020787 / 0.00521]^{1/4} = 0.0023092 / (\Psi)_{14.8}$$

in which $(\Psi)_{14.8}$ is the value of the wetted area constant for $Q_A = 0.0148\ cm^3cm^{-2}s^{-1}$. From Eq. 5

$$(E)_2 = (D_w/h)_2 / (e/h)_2 = 0.0023092 / [(0.481)\,(\Psi)_{14.8}] = 0.0048 / (\Psi)_{14.8}$$

The value of K is then obtained from Eq. 3. Substituting Eqs. 44a and 44b into Eq. 3, it results

$$K = E\,k_2L / k_1L = [0.0048 / (\Psi)_{14.8}]\,[8.4909 / (0.0044873 / (\Psi)_{14.8})] = 9.08252$$

The dimensionless filter depth α can be calculated using Eq. 20 as

$$(\alpha)_2 = (\alpha)_1\,[(\Psi)_2 / (\Psi)_1]\,[(Q_P)_1 / (Q_P)_2]\,[(\ell)_2 / (\ell)_1] = (\alpha)_1\,[(Q_P)_2 / (Q_P)_1]^{1/4}$$
$$[(Q_P)_1 / (Q_P)_2]\,[(\ell)_2 / (\ell)_1] = (\alpha)_1\,[(Q_P)_1 / (Q_P)_2]^{3/4}\,[(\ell)_2 / (\ell)_1] \qquad (46)$$

or directly from

$$(\alpha)_2 = [(\Psi)_1 k_1L / k_2L]\,[(\ell)_2 / (Q_P)_2]\,[\Psi)_2 / (\Psi)_1] =$$
$$[(\Psi)_1 k_1L / k_2L]\,[(\ell)_2 / (Q_P)_2]\,[(Q_P)_2 / (Q_P)_1]^{1/4} \qquad (47)$$

Substituting the numerical values into Eqs. 46 or 47 it results

$$(\alpha)_2 = 1.5499\,[0.020787 / 0.00521]^{3/4}\,165.00 / 60.96 = 11.8425$$

Table 3
The Predicted Substrate Removal Efficiencies η Under Various Experimental Conditions[a]

Inlet glucose concentration c_i mg dm^{-3}	Q_P, $cm^3cm^{-2}s^{-1}$	0.00521	0.0104	0.015	0.027	0.0658	0.078	0.087
	K	9.08252	7.62979	6.95342	5.98997	4.78497	4.57634	4.4457
	α	11.8425	7.05174	5.35800	3.44785	1.76767	1.55597	1.436
		η	η	η	η	η	η	η
23.45		—	—	—	0.9185	0.6778	0.6232	0.5886
87.93		—	0.9877	0.9466	0.7880	0.4888	0.4394	0.4096
175.85		—	0.9525	0.8559	0.6479	0.3744	0.3339	0.3100
293.08		0.9959	0.8675	0.7344	0.5224	0.2834	0.2499	0.2304
410.32		0.9771	0.7563	0.6080	0.4002	0.2063	0.1817	0.1674
527.55		0.9239	0.6366	0.4873	0.2945	0.1619	0.1426	0.1314
586.17		0.8859	0.5777	0.4407	0.2846	0.1462	0.1287	0.1186
703.40		0.7962	0.4858	0.3700	0.2386	0.1225	0.1079	0.0994
820.63		0.6977	0.4185	0.3185	0.2052	0.1053	0.0927	0.0854
937.87		0.6142	0.3674	0.2795	0.1800	0.0924	0.0813	0.0749
996.48		0.5793	0.3462	0.2639	0.1696	0.0870	0.0766	0.0706
1055.10		0.5481	0.3274	0.2490	0.1603	0.0823	0.0724	0.0667

[a]Depth of filter $\ell = 165$ cm; diameter of filter, $D = 35.4$ cm; liquid temperature, $T = 25\,°C$; specific surface, $S = 0.966\ cm^2cm^{-3}$; porosity, $\epsilon = 0.408$; $M = 8.491$; $A_1 = 0.1706\ cm^3g^{-3}$.

or

$$(\alpha)_2 = (0.0044873 / 8.4909)(165.00 / 0.00521)[0.00521 / 0.020787]^{1/4} = 11.8425$$

The other input data for the computer program are:

From Eq. 4

$$A_1 = (10^{-6})(k_3) = (10^{-6})(1.706)(10^5) = 0.1706\ cm^3g^{-1}$$

From Eq. 44b

$$M = k_2L = 8.4909$$

The necessary information for the biological system parameters were obtained using the equations derived. The computer input data were calculated for the given hydraulic loading rate of Q_A and the liquid temperature of 25 °C. They were written in Table 3 for different hydaulic loading rates of Q_A. The substrate removal efficiencies η and the values of (ηc_i) were then predicted performing the computer program for these input data. The resulting values were also written in Table 3 and plotted in Fig. 3 together with the measured values in Table 2. The predicted substrate removal ηc_i were in good agreement with the measured ones which indicates the validity of the developed equations and the assumption made to determine the wetted area constant Ψ.

SUMMARY AND CONCLUSIONS

The performance characteristics of the unsaturated granular media were described using Monod type biological kinetic and mass transfer concepts within microbial films. For this purpose computer techniques

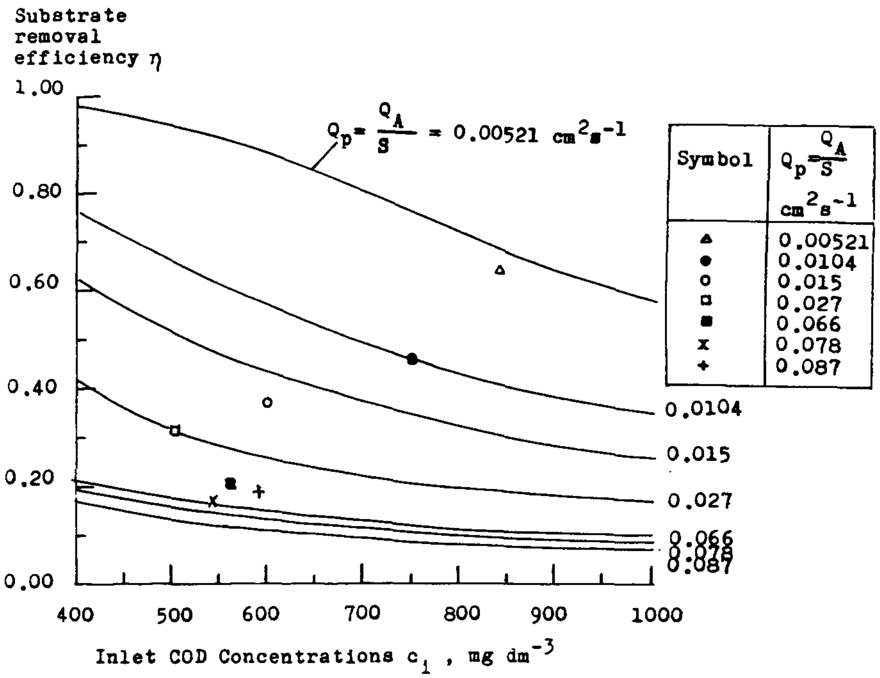

Fig. 3. Substrate removal efficiency η vs inlet COD concentrations c_i, for different hydraulic loadings in a trickling filter filled with spherical media at a temperature of 25°C.

were first developed for the numerical evaluation of the normalized biofilm mathematical model. The wetted surface area was expressed as a function of flow rate. The effect of flow rates on substrate utilization was then numerically evaluated using the computer program developed.

If the biological system parameters are known for a specific case, the theory enables one to transfer the results to another filter. The data from the literature were used for experimental confirmation of the equations derived. The biological parameters obtained from literature data were applied to predict effluent concentrations from two other experimental filters. For this purpose the input data for the computer program were determined using the developed equations and the effluent concentrations were calculated. The predicted results were found to be in good agreement with the measured values.

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